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Generalized Skew Derivation On **(σ, τ)**-Lie Ideals in Prime Rings

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Abstract: In the present paper, we extend some results concerning generalized skew derivation on (σ,τ)-Lie ideals of a prime rings.

Keywords: Prime ring, (σ,τ)-Lie ideal, generalized skew derivation, skew derivation.

1. INTRODUCTION

Throughout this paper, R is an associative ring, U a left ideal of R, $f : R \to$ R a generalized skew derivation associated with a nonzero skew derivation d and α an automorphsm of R. A ring R is said to be prime ring , if for any $x, y \in R, xRy = 0$ implies that either $x = 0$ or $y = 0$ and is called semiprime ring if for any $x \in R$, $xRx = 0$ implies $x = 0$. An additive mapping $d : R \to R$ is said to be a derivation of R if for any $x, y \in R$, $d(xy) = d(x)y + xd(y)$. By a skew derivation of R we mean an additive map d from R into itself which satisfies the rule $d(xy) = d(x)y + \alpha(x)d(y)$ for all $x, y \in R$ and α being an automorphism of R. For $\alpha = 1$ is the identity automorphism of R, d is known as a derivation of R. In particular, for a fixed $a \in R$, the mapping I_a : R → R given by $I_a(x) = [x, a]$ is a derivation called an inner derivation of R. The commutativity of prime rings with derivation was initiated by E. C. Posner [18]. Over the last two decades, a great deal of work has been done on this subject. A function $f_{a,b} : R \to R$ is called a generalized inner derivation if $f_{a,b}(x) = ax + xb$ for some fixed $a, b \in R$. It is straightforward to note that $f_{a,b}$ is a generalized inner derivation, then for any $x, y \in R$, $f_{a,b}(xy) = f_{a,b}(x)y + x[y, b] = f_{a,b}(x)y + xI_b(y)$ where I_b is an inner derivation. In view of the above observation, the concept of generalized derivation is introduced in [15] and [8] as follows: An additive mapping $f : R \to R$ is called a generalized derivation associated with a derivation d if $f(xy) = f(x)y + xd(y)$ for all $x, y \in R$.

An additive mapping $G : R \to R$ is called a generalized inner derivation if $G(x) = ax + xb$, for fixed $a, b \in R$. For such a mappings $G(xy) = G(x)y + x[y, b] = G(x)y + xI_b(y)$, for all $x, y \in R$. Motivated by the above observation, Bresar introduced the concept of generalized derivation as well as left multiplier mapping of R into R. The generalized derivation G of R is defined as an additive mapping $G : R \to R$ such that $G(xy) = G(x)y + xd(y)$ holds for any $x, y \in R$, where d is a derivation of R. So, every derivation is a generalized derivation, but the converse is not true in general. If $d = 0$, then we have $G(xy) = G(x)y$ for all $x, y \in R$, which is called a left multiplier mapping of R. Thus, generalized derivation generalizes both the concepts,. derivation on R. An additive mapping $G : R \to R$ is said to be a (right) generalized skew derivation of R if there exists a skew derivation d of R with an associated automorphism α such that $G(xy) = G(x)y + \alpha(x)d(y)$ holds for all $x, y \in R$. For any two elements $x, y \in R$, $[x, y]$ will denote the commutator element $xy - yx$ and $[x, y]_{\sigma, \tau} = x\sigma(y) - \tau(y)x$ and $xoy = xy + yx$. We use extensively the following basic commutator identities:

i. $[xy, z] = x[y, z] + [x, z]y$
ii. $[x, yz] = [x, y]z + y[x, z]$
iii. $[xy, z]_{\sigma, \tau} = x[y, z]_{\sigma, \tau} + [x, \tau(z)]y = x[y, \sigma(z)] + [x, z]_{\sigma, \tau}y$
iv. $[x, yz]_{\sigma, \tau} = \tau(y)[x, z]_{\sigma, \tau} + [x, y]_{\sigma, \tau}\sigma(z)$.

Let U be an additive subgroup of R. The definition of (σ, τ)-Lie ideal of R is given in [16] as follows:

i. U is a (σ, τ)-right Lie ideal of R if $[U,R]_{\sigma, \tau} \subset U$.
ii. U is a (σ, τ)-left Lie ideal of R if $[R,U]_{\sigma, \tau} \subset U$
iii. U is a (σ, τ)-Lie ideal of R, if U is both a (σ, τ)-right Lie ideal and (σ, τ)-left Lie ideal of R..

One may observe that the concept of generalized derivation includes the concept of derivations and generalized inner derivations, also of the left multipliers when $d = 0$. Hence it should be interesting to extend some results concerning these notions to generalized derivations. Some recent results were shown on generalized derivation in [8], [15] and [1].

Furthermore, some authors have also studied generalized derivation in the theory of operator algebras and C^*-algebras (see for example [15]). On the other hand, in [10, Definition 1], Golbasi and Kaya introduced the notation of right generalized derivation and left generalized derivation with associated derivation *d* as follows:

An additive mapping $f : R \rightarrow R$ is said to be right generalized derivation with associated derivation *d* if $f(xy) = f(x)y + xd(y)$ for all $x, y \in R$ and *f* is said to be left generalized derivation with associated derivation *d* if $f(xy) = d(x)y + xf(y)$ for all $x, y \in R$. An additive map *f* is said to be a generalized derivation with associated derivation *d* if it is both a left and right generalized derivation with associated derivation *d*. Of course, every derivation is generalized derivation and also, the definition of generalized derivation given in Bresar [8] is a right generalized derivation with associated derivation *d* according to above definition. In this context, we mention the definition of generalized derivation that means two sided generalized derivation.

In [1], Argac and Albas proved that if a prime ring *R* has (d,α), (g,β) nonzero generalized derivations such that $ad(x) = g(x)a$ for all $x \in R$, then one of the following possibilities holds; (i) $a \in C$ (extended centroid). (ii) There exist $p,q \in Q_r\,(R_C)$ (a right Martindale ring of quotients) such that $\alpha(x) = [x, p]$, $\beta(x) = [q, x]$, $qa \in C$, $p = \lambda a$, where $\lambda \in C$, for all $x \in R$. And the same result extended in [9], Oznur Golbasi and Emine Koc for generalized derivations on (σ, τ)-left Lie ideal of *R*.

In [11], Herstein showed that if *R* is a prime ring of characteristic different from two and *d* is a nonzero derivation such that $d(R) \subset Z$, then *R* must be commutative. Several authors investigated this result for Lie ideals or (σ, τ) Lie ideals of a prime ring admitting derivation or generalized derivation (see [7], [6], [5], [9]). Oznur Golbasi and Emine Koc proved extend some results on generalized derivations of semiprime rings in [2] and corresponding results for (σ, τ)-Lie ideal of a prime ring with generalized derivation [10]. Later on so many authors are extended the results for generalized (α, β)-derivation on ideals, Lie ideals in prime rings, semi prime rings. In [14], the author given some results on generalized (α, β)-derivation on Lie Ideals of σ-prime rings. Now our aim is to extend [10], Oznur Golbasi and Emine Koc the results for generalized skew derivation on (σ, τ)-Lie ideal of a prime ring.

Throughout the present paper, we assume that *R* be a prime ring with characteristic not two, α, β, σ and τ are automorphisms and *U* a nonzero (σ, τ)-Lie ideal of *R*. We denote a generalized (α, β)-derivation $f : R \rightarrow R$ with a non-zero (α, β)-derivation *d* of *R* by (f, d). If $d = 0$, then $f(xy) = f(x)\,\alpha(y)$ for all $x,y \in R$ and there exists $q \in Q_r(R_C)$ such that $f(x) = qx$ for all $x \in R$ by [15, Lemma 2]. So, we assume that $d \neq 0$.

An additive mapping $f : R \rightarrow R$ is said to be right generalized (α, β) −derivation with associated non-zero (α, β)-derivation *d* if

(1.1) $f(xy) = f(x)\,\alpha(y) + \beta(x)d(y)$ for all $x, y \in R$ and *f* is said to be left generalized (α, β) −derivation with associated derivation *d* if

(1.2) $f(xy) = d(x)\,\alpha(y) + \beta(x)\,f(y)$ for all $x, y \in R$.

f is said to be a generalized derivation with associated derivation *d* if it is both a left and right generalized derivation with associated derivation *d*.

2. PRELIMINARIES :

Lemma 1. [4, Lemma 3] *Let R be a prime ring with* char $R \neq 2$, $a \in R$ *and* $aU = 0$ (*or* $Ua = 0$).

i. *If U is a* (σ, τ)*-right Lie ideal of R, then* $a = 0$ *or* $U \subset C_{\sigma, \tau}$.
ii. *If U is a* (σ, τ)*-left Lie ideal of R, then* $a = 0$ *or* $U \subset Z$.

Lemma 2. [3, Lemma 6] *Let R be a prime ring with* char $R \neq 2$ *and U a*(σ, τ)*-left Lie ideal of R. Suppose there exists* $a \in R$ *such that* $[a, U] = 0$. *Then* $a \in Z$ *or* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Lemma 3. [13, Theorem 2] *Let R be a prime ring with* char R $\neq 2$ *and U a non-central* (σ, τ)*-left Lie ideal of R. Then there exist a nonzero ideal M of R such that* $[R,M]_{\sigma, \tau} \subset U$ *and* $[R,M]_{\sigma, \tau} \not\subseteq C_{\sigma, \tau}$ *or* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

3. MAIN RESULTS

Theorem - 1 : *Let R be a prime ring with* char $R \neq 2$, *f be a generalized skew derivation of R with a nonzero skew derivation d and an automorphism* α *of R and U a non-central* (σ, τ)*-left Lie ideal of R. If* $f(U) = 0$, *then* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Proof : Suppose to the contrary that $\sigma(u) + \tau(u) \notin Z$ for some $u \in U$. By Lemma 3, there exists a nonzero ideal *M* of *R* such that $[R,M]_{\sigma, \tau} \subset U$, but $[R,M]_{\sigma, \tau} \not\subseteq C_{\sigma, \tau}$. For any $x \in R$ and $m \in M$, $[x, m]_{\sigma, \tau}\,\sigma(m) = [x\sigma(m),m]_{\sigma, \tau} \in U$. Then $0 = f([x, m]_{\sigma, \tau}\,\sigma(m)) = f([x, m]_{\sigma, \tau})\sigma(m)) + \alpha([x, m]_{\sigma, \tau})d(\sigma(m))$ and so

(3.1) $\alpha([x, m]_{\sigma, \tau})d(\sigma(m)) = 0$ for all $x \in R$, $m \in M$.

Replacing *x* by *xy*, $y \in R$ in (3.1) and applying (3.1), we get $0 = \alpha([xy, m]_{\sigma, \tau})d(\sigma(m)) = \alpha(x[y, m]_{\sigma, \tau}\,d(\sigma(m)) + \alpha([x, \tau(m)]y)d(\sigma(m))$. That is, $\alpha([x, \tau(m)])Rd(\sigma(m)) = 0$ for all $x \in R$, $m \in M$. Since *R* is a prime ring, it follows that $\alpha([x, \tau(m)]) = 0$ or $d(\sigma(m)) = 0$ for all $m \in M$. If $\alpha([x, \tau(m)]) = 0$, since α is an automorphism of R, applying α^{-1} on both sides we get $[x, \tau(m)] = 0$, hence $m \in Z$. We set $K = \{m \in M \mid m \in Z\}$ and $L = \{m \in M \mid d(\sigma(m)) = 0\}$. Clearly each of K and L is additive subgroup of M. Moreover, M is the set-theoretic union of K and L. But a group can not be the set-theoretic union of its two proper subgroups, hence $K = M$ or $L = M$. In the former case, $M \subset Z$ which forces R to be commutative. This is impossible because of $U \not\subseteq Z$. In the latter case, $d(\sigma(M)) = 0$. Since R is a prime ring and $\sigma(M)$ a nonzero ideal of R, we get $d = 0$, a contradiction. This completes the proof. □

Theorem - 2 : *Let R be a prime ring with* char R $\neq 2$, *f be a generalized skew derivation of R with a nonzero skew derivation d and α an automorphism α of R and U a noncentral (σ, τ)-left Lie ideal of R. If* $d(Z) \neq 0$ *and* $[f(U), a]_{\sigma,\tau} = 0$, *then* $a \in Z$ *or* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Proof: Choose $z \in Z$ such that $d(z) \neq 0$. It is easily seen that $d(z) \in Z$. For all $x \in R$, $u \in U$, we get

$0 = [f([x, u]_{\sigma,\tau} z), a]_{\sigma,\tau}$

$= [f([x, u]_{\sigma,\tau}) z + \alpha([x, u]_{\sigma,\tau}) d(z), a]_{\sigma,\tau}$

$= [f([x, u]_{\sigma,\tau}), a]_{\sigma,\tau} z + f([x, u]_{\sigma,\tau})[z, \sigma(a)] + [\alpha([x, u]_{\sigma,\tau}), a]_{\sigma,\tau} d(\alpha(z)) + \alpha([x, u]_{\sigma,\tau})[d(\alpha(z)), \sigma(a)]$

and so $\alpha([x, u]_{\sigma,\tau}, a]_{\sigma,\tau}) d(z) = 0$ for all $x \in R$, $u \in U$. Since R is prime and $0 \neq d(z) \in Z$, we see that

(3.2) $\alpha([x, u]_{\sigma,\tau}, a]_{\sigma,\tau}) = 0$ for all $x \in R$, $u \in U$.

Substituting $x\sigma(u)$ for x in (3.2) and using this equation, we obtain $\alpha([x,u]_{\sigma,\tau})\sigma([u, a]) = 0$ for all $x \in R$, $u \in U$. Now, taking xy instead of x in the last equation, we obtain $[R, \tau(u)]\, R\, \sigma([u, a]) = 0$ for all $u \in U$. Since R is a prime ring, it follows either $u \in Z$ or $[u, a] = 0$ for all $u \in U$. By a standard argument one of these must hold for all $u \in U$. If $u \in Z$ for all $u \in U$, then $U \subset Z$, and so $\sigma(u) + \tau(u) \in Z$ for all $u \in U$. If $[U, a] = 0$, then $a \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$ by Lemma 2. Thus the proof is completed.

Theorem – 3 : *Let R be a prime ring with* char $R \neq 2$, *(f, d), (g, h) two generalized skew derivations of R and U a noncentral (σ, τ)-left Lie ideal of R. If* $f(u)v = \alpha(u)g(v)$ *for all* $u, v \in U$, *then* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Proof : Suppose that $\sigma(u) + \tau(u) \notin Z$ for some $u \in U$. Then there exists a nonzero ideal M of R such that $[R,M]_{\sigma,\tau} \subset U$ and $[R,M]_{\sigma,\tau} \not\subseteq C_{\sigma,\tau}$ by Lemma 3. For any $x \in R$ and $m \in M$, $\tau(m)[x, m]_{\sigma,\tau} = [\tau(m)x, m]_{\sigma,\tau} \in U$.
Taking $\tau(m)[x, m]_{\sigma,\tau}$ instead of u in the hypothesis, we get

$$f(\tau(m)[x, m]_{\sigma,\tau})v = \alpha(\tau(m)[x, m]_{\sigma,\tau})\, g(v),$$
$$d(\tau(m))[x, m]_{\sigma,\tau} v + \alpha(\tau(m)) f([x, m]_{\sigma,\tau})v = \alpha(\tau(m)[x, m]_{\sigma,\tau})\, g(v).$$

That is,

$d(\tau(m))[x, m]_{\sigma,\tau} v + \alpha(\tau(m))\, \alpha([x, m]_{\sigma,\tau}) g(v) = \alpha(\tau(m)\, \alpha([x, m]_{\sigma,\tau})\, g(v)$.
Hence we get, $d(\tau(m))[x, m]_{\sigma,\tau} v = 0$ for all $m \in M$, $v \in U$, $x \in R$. That is $d(\tau(m))[x, m]_{\sigma,\tau} U = (0)$ for all $m \in M$, $x \in R$. By Lemma 1, we obtain that

(3.3) $d(\tau(m))[x, m]_{\sigma,\tau} = 0$ for all $m \in M$, $x \in R$.

Replacing x by xy, $y \in R$ in (2.3) and using (2.3), we have $d(\tau(m))x[y, \sigma(m)] = 0$ and so $d(\tau(m))R[y, \sigma(m)] = 0$ for all $m \in M$, $y \in R$. Since R is a prime ring, it follows that $m \in Z$ or $d(\tau(m)) = 0$ for all $m \in M$. Let $L = \{m \in M \mid m \in Z\}$ and $K = \{m \in M \mid d(\tau(m)) = 0\}$. By the same method in Theorem-1, we get $d = 0$, a contradiction. This completes the proof.
An immediately results of Theorem-3 we give the following corollaries.

Corollary – 1. *Let R be a prime ring with* char $R \neq 2$, *(f, d), (g, h) two generalized skew derivations of R and U a noncentral (σ, τ)-left Lie ideal of R. If* $f(u)u = \alpha(u)g(u)$ *for all* $u \in U$, *then* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

In particular if we take $f = g$, then we have the following corollary, which is a generalization of [13, Theorem] for the case when characteristic of underlying ring is different from two.

Corollary - 2. *Let R be a prime ring with* char $R \neq 2$, *f is a generalized skew derivation of R and U a noncentral (σ, τ)-left Lie ideal of R. If* $[\alpha(u), f(u)] = 0$ *for all* $u \in U$, *then* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Corollary - 3. *Let R be a prime ring with* char $R \neq 2$, *d, h two nonzero skew derivations of R and U a noncentral (σ, τ)-left Lie ideal of R. If* $d(u)v = \alpha(u)h(v)$ *for all* $u, v \in U$, *then* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Theorem - 4. *Let R be a prime ring with* char $R \neq 2$, *U a nonzero (σ, τ)-left Lie ideal of R. Let* $a, b \in R$ *and* $f : R \to R$ *be a mapping such that* $f(x) = xa - bx$ *for all* $x \in R$. *If* $f(U) \subset U$ *and* $f(U) \subset Z$, *then* $\sigma(u) + \tau(u) \in Z$ *for all* $u \in U$.

Proof : By the hypothesis, for all $u \in U$, we have $f(u) = ua - bu \in Z$. Commuting this element by u, we obtain that,

(3.4) $u[a, u] = [b, u]u$ for all $u \in U$.

A linearization of (3.4) yields that $u[a, v] + v[a, u] = [b, v]u + [b, u]v$. Taking $f(u)$ instead of u in the above equation, we find that $f(u)[a, v] + v[a, f(u)] = [b, v]f(u) + [b, f(u)]v$. Using $f(u) \in Z$ in the last equation, we have

(3.5) $f(u)([a, v] - [b, v]) = 0$ for all $u, v \in U$.

Using the primeness of R and $f(u) \in Z$ in (2.5), we conclude that $f(U) = 0$ or $[a - b, U] = (0)$. If $[a - b, U] = (0)$, then $a - b \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$ by

Lemma 2. Now, we assume that $a - b \in Z$. From (2.4), we obtain

(3.6) $u^2a - uau = ubu - bu^2$ for all $u \in U$.

Let $a - b = \alpha$, $\alpha \in Z$. Writing $a = b + \alpha$ in (2.6), we have $u^2b + u^2\alpha - ubu - u\alpha u = ubu - bu^2$ and $u^2b + u^2\alpha + bu^2 = 2ubu + u\alpha u$. Using $\alpha \in Z$ in the last equation, we get $u^2b + bu^2 - 2ubu = 0$, and $u^2b - ubu = ubu - bu^2$ and so $u[u, b] = [u, b]u$. That is $[u,[u, b]] = 0$ for all $u \in U$. This yields that $[u, d_b(u)] = 0$, where $d_b : R \to R$, $d_b(x) = [x, b]$ is an inner derivation of R. Therefore $\sigma(u)+\tau(u) \in Z$ for all $u \in U$ or $d_b = 0$ by Corollary 2. If $d_b = 0$, then $b \in Z$. We have $f(u) = ua - bu = u(a - b) \in Z$, by the hypothesis. Since $a - b \in Z$ and R is a prime ring, we obtain that $U \subset Z$ or $a - b = 0$. Now, we assume that $a = b$. Using $b \in Z$, we get $f(x) = xa - bx = xb - bx = 0$ for all $x \in U$. As a result $f = 0$, and so $f(U) = 0$. Hence $\sigma(u) + \tau(u) \in Z$ for all $u \in U$ according to Lemma 4. This completes the proof. □

Corollary 4. *Let R be a prime ring with* char $R \neq 2$ *,U a nonzero (σ, τ)-Lie ideal of R and $a \in R$. If* $[U, a]_{\sigma,\tau} \subset$ *Z, then $a \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Proof : Let f be a mapping such that $f(x) = [x, a]_{\sigma,\tau} = x\sigma(a) - \tau(a)x$ for all $x \in R$. Since U is a (σ,τ)- Lie ideal, we have $f(U) \subset U$. By Theorem 2, we get $\sigma(u) + \tau(u) \in Z$ for all $u \in U$. □

Corollary 5. *Let R be a prime ring with* char $R \neq 2$, *U a nonzero (σ, τ)-Lie ideal of R. If $[U,U]_{\sigma,\tau} \subset Z$, then $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

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